B.A./B.Sc. Part III (Honours) Examination, 2021 (1+1+1)
Subject: Mathematics
Paper VI

Time: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any four questions: 4×5 = 20

(a) Explain the concept of momental ellipsoid and find the momental ellipsoid at the centre of an elliptic plate. [1+4]

(b) Obtain the equation of a catenary of uniform strength. [5]

(c) A uniform rod of length *2a* is placed with one end in contact with a horizontal table and is at an inclination α to the horizon and is allowed to fall. When it becomes horizontal, show that its angular velocity is $\sqrt{\frac{3g}{2a}\sin\alpha}$, whether the plane be perfectly smooth or perfectly rough. [5]

(d) Prove that for any given system of forces, the quantities $\mathbf{X^2 + Y^2 + Z^2}$ and $\mathbf{LX + MY + NZ}$ are invariable whatever origin, or base point and axes are chosen. [5]

(e) Explain how to determine the resultant thrust on a curved surface bounded by a plane curve exposed to pressure of heavy fluid at rest under gravity. [5]

(f) A given volume V of a liquid is acted on by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$ per unit mass at (x, y, z) parallel to the axes. Find the equation of free surface. [5]

2.Answer any three questions: 3×10 = 30

(a) (i) State and verify the principle of conservation of moment of momentum. [5]

(ii) Show that the kinetic energy of a rigid body rotating about a fixed point O is given by $T = \frac{1}{2}\left(A\omega_1^2 + B\omega_2^2 + C\omega_3^2\right)$, where *A, B, C* are the principal moments of inertia at O and ω_1, ω_2, ω_3 are the components of angular velocity along the principal axes respectively. [5]

(b) A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane is $\frac{1}{3}\tan\alpha$ so that it may roll without sliding. [10]

(c) (i) Explain the concept of virtual work in statics and write down the principle of virtual work for a system. [3+2]

(ii) Two forces ***P, Q*** act along the straight lines whose equations are $y = x\tan\alpha$, $z = c$ [5]

and $y = -x \tan\alpha$, $z = -c$ respectively. Show that their central axis is given by $y = x\frac{P-Q}{P+Q}\tan\alpha, \frac{z}{c} = \frac{P^2 - Q^2}{P^2 + 2PQ\cos 2\alpha + Q^2}$.

(d) (i) Establish the energy test of stability of equilibrium of a body with one degree of freedom. [5]

(ii) A thin hollow cone, with a base, floats completely immersed in water whenever it is placed; show that the vertical angle is $2\sin^{-1}\frac{1}{3}$. [5]

(e) (i) A circular area of radius *a* is immersed in a homogeneous liquid with its plane vertical and centre at a depth *h* ; find the depth of the centre of pressure. [5]

(ii) What is an adiabatic change of state? Derive the relation pv^{γ} = constant for adiabatic expansion of a compressible fluid, where the symbols are to be explained by you. [1+4]