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B.A. 6th Semester (General) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BAMATH6GE2 (Generic Elective)

(Geometry and Vector Calculus)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Find the vertex, focus, latus rectum and axis of the parabola $3x^2 - 9x - 5y - 2 = 0$. [2+1+1+1]

(b) Find the equation of the hyperbola whose focus is (2,3), directrix is the line $x + 2y = 1$ and eccentricity is $\sqrt{3}$. [5]

(c) Find the equation of the ellipse whose axes are the axes of co-ordinates, eccentricity is $\sqrt{2}/5$ and which passes through the point (-3,1). [5]

(d) Find the equation of the sphere which passes through the points (1,-3,4), (1,-5,2), (1, -3,0) and whose centre lies on the plane $x + y + z = 0$. [5]

(e) If $\vec{r} = 3t\vec{\imath} + 3t^2\vec{\jmath} + 2t^3\vec{k}$, then find $\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}$. [5]

(f) If $\vec{\alpha} = 3t^2\vec{\imath} + t\vec{\jmath} - t^3\vec{k}$ and $\vec{\beta} = \sin t\,\vec{\imath} - 2\cos t\,\vec{\jmath}$ then find $\frac{d}{dt}(\vec{\alpha} \times \vec{\beta})$ [5]

(g) If $\vec{r} = (3x^2 - xy^2)\vec{\imath} + (e^{xy} - x\sin y)\vec{\jmath} + (y^2 cosx)\vec{k}$ then find $\frac{\partial \vec{r}}{\partial x}$ and $\frac{\partial \vec{r}}{\partial y}$. [5]

(h) Find $div\ grad\ \varphi$, where $\varphi = 2x^2y^3z^4$. [5]

2. Answer any three questions: 10×3 = 30

(a) Reduce the equation $5x^2 - 6xy + 5y^2 + 22x - 26y + 29 = 0$ to the normal form and then find the nature of the conic. [10]

(b) (i) Find the equation of the cylinder whose generators are parallel to the straight line with direction ratios -2,1,3 and whose guiding curve is the ellipse $2x^2 + y^2 = 1, z = 0$. [6]

(ii) Find the equation to the sphere which passes through the circle $x^2 + y^2 + z^2 = 9,\ x + y - 2z = 4$ and the origin. [4]

(c) (i) Find the directional derivative of $f = xy + yz + zx$ in the direction $(\vec{\imath} + 2\vec{\jmath} + 2\vec{k})$ at the point (1,2,0). [5]

(ii) If r is the distance of $P(x,y,z)$ from the origin and $\vec{r}$ is the position vector of P relativeto the origin then find $grad\ r$ and $div\ \vec{r}$ [3+2]

(d) (e) Show that $curl\{f(r)\vec{r}\} = \vec{0}$, where $\vec{r}$ is the position vector , $r = |\vec{r}|$ and $f(r)$ is differentiable. [10]

(e) (i) Verify that $curl\ (grad\ f) = \vec{0}$, where $f = x^2y + 2xy + z^2$. [5]

(ii) Prove that $\vec{\nabla}.(r^3\vec{r}) = 6r^3$. [5]