B.A./B.Sc. 6th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH6DSE31
(Mathematical Modelling)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Discuss transient and steady states of a queuing system. [2+3]

(b) Discuss biotic and abiotic factors of an ecosystem. [5]

(c) Show that the equilibrium (x^*, y^*) with $x^* > 0, y^* > 0$ of the predator-prey model, [5]

$$\frac{dx}{dt} = Ax\left(\frac{y}{y+a} - \frac{b}{a+b}\right)$$
$$\frac{dy}{dt} = y(1 - y/k) - \frac{Axy}{y+a}$$

is unstable if $k > a + 2b$ and asymptotically stable if $b < k < a + 2b$ [A being a constant].

(d) Discuss Gompertz population model. [5]

(e) Discuss density dependent growth model. [5]

(f) Explain the following: [2+3]

(i) Poisson axioms of departures of a queueing system,

(ii) Service discipline of a queueing system.

(g) Find the non-negative equilibrium point of a population governed by, [3+2]

$$x_{n+1} = \frac{2x_n^2}{x_n^2 + 2}$$

and investigate the stability.

(h) Write down the logistic model for a single-species population. Hence explain the concepts of carrying capacity and intra-species competition. [2+3]

2. Answer any three questions: 10×3 = 30

(a) Consider the queueing model $(M/M/1):(\infty/FCFS/\infty)$. Find the following: [5×2]

(i) probability of queue size greater than or equal to N.

(ii) expected number of customers in the system.

(iii) expected number of customers in the queue.

(iv) average length of non-empty queue.

(v) expected waiting time in the system.

(b) Discuss Malthus model of population growth. Solving Malthus growth equation with a given initial condition, show that a population obeying this equation undergoes exponential growth or decay. Determine the population doubling time for Malthus model. What are the drawbacks of Malthus model? Describe a model in which these drawbacks have been overcome. [10]

(c) Obtain the maximum likelihood estimator of μ, where μ and σ (known) are mean and standard deviation of a normal population respectively. Show that this estimator is unbiased. [8+2]

(d) Consider the following system: [2×5]

$$\frac{dx}{dt}=F(x,y),\ \frac{dy}{dt}=G(x,y)$$

where F and G are continuously differentiable on $\mathbb{R}^2$. Define an equilibrium point of the system. Define stability and asymptotic stability of an equilibrium point. Linearize the system about an equilibrium point and write down the corresponding community matrix. State conditions for asymptotic stability of the equilibrium point. What are the conditions for the equilibrium point to be a stable spiral?

(e) Discuss the model, [4+3+3]

$\frac{dx}{dt}=rx(1-x/k)(x/k_0-1)$,

where $0<k_0<k$ and r is the growth rate. Find all the limits of solutions with $x(0)>0$ as $t\to\infty$ and find the set of initial values corresponding to each limit.

B.A./B.Sc. 6th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH6DSE32
(Industrial Mathematics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) What do Hounsfield units measure? Discuss briefly. [5]

(b) What is a Sinogram Radon transform? Discuss briefly. [5]

(c) What is convolution back projection? Discuss briefly. [5]

(d) Briefly illustrate the effects of angular undersampling. [5]

(e) Draw a case of Linogram reconstruction. [5]

(f) Applying the scaling property of the ideal ramp filter find out fan-beam reconstruction formula. [5]

(g) Compute weighted projections for each parametric change. [5]

(h) Demonstrate equiangular case (3rd generation multi-slice CT). [5]

2. Answer any three questions: 10×3 = 30

(a) Deduce the Laplacian property of the Radon transform. [10]

(b) Obtain Radon transform property for an affine transformation of the object f(x, y). [10]

(c) Suppose that a Hanning window is applied to the ramp filter. Using the properties of Fourier series, find the impulse response analytically. [10]

(d) Give an example of two random variables that are individually Gaussian distributed but their joint distribution is not Gaussian. [10]

(e) Set an example of back projection. Explain with graphs. [5+5]

B.A./B.Sc. 6th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH6DSE33
(Group Theory II)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Prove that commutator subgroup G' of a group G is normal in G. Show that G' is a characteristic subgroup of G. [2+3]

(b) Let G be group of order p^2, where p is prime. Prove that G is commutative. [5]

(c) Find the conjugacy classes in S_3 and write down the class equation. [5]

(d) Find $|\mathrm{Aut}(Z_2 \times Z_2)|$. [5]

(e) Prove that no group of order 56 is simple. [5]

(f) Let G be a group of order $2m$, where m is an odd integer. Prove that G has a normal subgroup of order m. [5]

(g) Let G be a group which acts on a nonempty set X, and for $x, y \in X$, define $x \sim y$ if there is a $g \in G$ such that $g \cdot x = y$. Prove that $\sim$ is an equivalence relation on X, and find the equivalence class of $x \in X$. [5]

(h) Let G be a finite group and H be a subgroup of G of index p, where p is the smallest prime dividing $|G|$. Prove that H is normal in G. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) Suppose G is a finite group. If p divides the order of G and p is prime, prove that G has an element of order p. [6]

(ii) Determine the number of elements of order 5 in $Z_{25} \times Z_5$. [4]

(b) (i) Let G and H be two finite cyclic groups. Prove that G×H is cyclic if and only if |G| and |H| are relatively prime. [6]

(ii) Prove that every subgroup of a cyclic group G is a characteristic subgroup of G. [4]

(c) (i) Suppose G is cyclic group of order n. Prove that Aut (G) is a group of order $\varphi(n)$, where $\varphi(n)$ is the number of positive integers less than n and prime to n. [4]

(ii) Let G = $\{z \in \mathbb{C} : |z| = 1\}$ be a group with respect to usual multiplication. Prove that the mapping $\phi : G \times \mathbb{C} \rightarrow \mathbb{C}$, defined by $\phi(g,a) = g.a$ is a group action. Find orbits for $z \in \mathbb{C}$. [3+3]

(d) (i) Show that $Inn(G) \cong \frac{G}{Z(G)}$, where $Z(G)$ is the centre of the group G. [4]

(ii) Let p be a prime. Prove that $Zp \times Zp$ has exactly $p+1$ subgroups of order p. [6]

(e) (i) Is Z×Z is cyclic? Justify your answer. [4]

(ii) Show that any group of order 255 is isomorphic to Z_{255}. [6]