B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH5CC11
(Partial Differential Equations and Applications)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Form a quasi linear first order partial differential equation from the given relation, [5]
$z = f(x-3y) + g\big(log(x-3y)\big) + h(3x-y)$.

(b) Solve the equation, [5]
$$xp - yq = \frac{y^2-x^2}{z},$$
given that $z(x_0(t), y_0(t)) = t$ on the curve $\gamma: x = x_0(t) = 2t, y = y_0(t) = t, t > 0$.

(c) Solve: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$. [5]

(d) If $z(x,y)$ be the solution of $xp + q = 1$ with initial condition $z(x,0) = logx$, then find $z(e,1)$. [5]

(e) Determine the region where the partial differential equation, [5]
$$(x^2+y^2-1)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\partial y} + (x^2+y^2-1)\frac{\partial^2 u}{\partial y^2} = 0$$
is hyperbolic, parabolic or elliptic .

(f) Consider partial differential equation of the form: [5]
$$ar + bs + ct + f(x,y,z,p,q) = 0 \text{ with } b^2 - 4ac > 0.$$
Describe the steps of reducing the above equation into its canonical form.

(g) Obtain the solution of the diffusion equation [5]
$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}, K > 0, t > 0, a < x < b$$
subject to the conditions

i) $u(x,t)$ remains finite as $t \to \infty$

ii) $u_x(a,t) = u_x(b,t) = 0, t \geq 0$

iii) $u(x,0) = f(x), a \leq x \leq b$.

(h) Solve, $(y+z)p - (x+z)q = x - y$. [5]

2. Answer any three questions: $10\times3 = 30$

(a) (i) Express the Laplace equation $\nabla^2 u = 0$ in cylindrical coordinates. [6]

(ii) Find the equation of the integral surface of the partial differential equation [4]

$2y(z-3)p + (2x-z)q = y(2x-3)$

which passes through the circle

$z = 0, x^2 + y^2 = 2x.$

(b) (i) Reduce the partial differential equation $y^2 \frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to its canonical form. [6]

(ii) Using the method of characteristics solve the Cauchy problem: [4]

$pz + q = 1,$

given that $z(x_0(t), y_0(t)) = t/2$ on the curve $\gamma: x = x_0(t) = t, y = y_0(t) = t, 0 \le t \le 1.$

(c) (i) Solve: $xp - yq = z$ with initial condition $z(x,0) = f(x), x \ge 0.$ [5]

(ii) Solve $(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x+y).$ [5]

(d) (i) A tightly stretched string of length π is held fixed at its ends $x = 0$ and $x = \pi$ and is subjected to an initial displacement [6]

$u(x,0) = u_o sin2x, 0 \le x \le \pi$

and velocity

$u_t(x,0) = v_o sinx, 0 \le x \le \pi$

If the displacement $u(x,t)$ satisfies the equation

$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0,\ 0 < x < \pi, t > 0,$

determine $u(x,t)$ by D' Alembert's method.

(ii) Prove that solution of $\frac{\partial^2 z}{\partial x^2} + z = 0$, with $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$, is $z = sinx + e^y cosx.$ [4]

(e) (i) Solve the partial differential equation: $\frac{\partial^2 z}{\partial x \partial y} = xy^2.$ [6]

(ii) Solve by the method of separation of variable [4]

$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0.$