B.A./B.S.c 6th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH6DSE41
(Bio Mathematics)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) What is the carrying capacity in logistic growth model? Explain graphically. [2+3]

(b) Explain the concept of a single population harvesting model with logistic growth. [5]

(c) Explain the nature of the following model, [5]

$$\frac{dN_1}{dt} = r_1N_1[1-\frac{N_1}{K_1}-b_{12}\frac{N_2}{K_1}]$$

$$\frac{dN_2}{dt} = r_2N_2[1-\frac{N_2}{K_2}-b_{21}\frac{N_1}{K_2}],$$

where r_1, K_1, r_2, K_2, b_{12} and b_{21} are all positive constants and have their usual meanings.

(d) Obtain the condition under which the following model will have a positive interior equilibrium point. [5]

$$\frac{dx}{dt} = x(1-x-py)$$

$$\frac{dy}{dt} = y(1-y-qx)$$, where p, $q>0$.

(e) Define the equilibrium solution of a difference equation. Find equilibrium solution of difference equation $x_{t+1} = rx_t(1-x_t)$. [3+2]

(f) Explain the concept of a simple discrete prey predator model. [5]

(g) Obtain the Routh-Hurwitz criteria for a cubic monic polynomial to have all negative or negative real roots. [5]

(h) Explain the concept of a continuous age-structured model. [5]

2. Answer any three questions: 10×3 = 30

(a) Consider the following model for bacterial growth in a chemostat, [4+6]

$$\frac{db}{dT} = b[\frac{k_{max}n}{k_n+n}-D]$$

$$\frac{dn}{dT} = D(n_0-n)-\beta\frac{k_{max}nb}{k_n+n}.$$

Reduce this system into its dimensionless form and hence discuss the stability of its equilibrium points.

(b) In the following SIRS epidemic model, analyze the phase plane by taking the parameters with their usual meaning, [4+6]

$$\frac{dS}{dt} = -\frac{\beta}{N} SI - \nu(N - S - I),$$

$$\frac{dI}{dt} = \frac{\beta}{N} SI - \gamma I.$$

Considering two cases, $R_0 > 1$ and $R_0 \le 1$, find the equilibrium points and determine the conditions for their local asymptotic stability, where R_0 is the basic reproduction number.

(c) Consider the following system, [3+5+2]

$$\frac{dx}{dt} = x(4 - x - y)$$

$$\frac{dy}{dt} = y(8 - 3x - y)$$

representing the change in densities of two competing species x and y. Find corresponding equilibrium points. Determine the stability of each equilibrium and state their nature.

(d) Solve the following initial value problem by the the method of characteristics, [6+2+2]

$$u_t + \nu u_x = 0,\ t \in (0, \infty),\ x \in (-\infty, \infty),$$

$$u(0, x) = \phi(x),\ x \in (-\infty, \infty).$$

Hence define the characteristic curves and the traveling wave solution.

(e) Define the Nicholson-Bailey model with density dependence in the host parasite population. Find its simplified form by changing the state variables and hence find implicit equations satisfied by the nonzero equilibrium of the simplified system. [2+4+4]

B.A./B.S.c 6th Semester (Honours) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMH6DSE42

(Differential Geometry)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Define signed curvature of a unit speed plane curve and prove that it is the rate at which the tangent vector of the curve rotates. [1+4]

(b) If γ is a space curve, then prove that its torsion is given by [5]

$$-\tau = \frac{(\ddot{\gamma} \times \dot{\gamma}).\dddot{\gamma}}{\|\ddot{\gamma} \times \dot{\gamma}\|^2}$$

where '×' indicates vector product and $\dot{\gamma} = \frac{d}{dt}(\gamma)$.

(c) Compute the torsion of the circular helix $\gamma(\theta) = (\mathrm{a}\cos\theta,\ a\sin\theta,\ b\theta)$, where $-\infty < \theta < \infty$ and a, b are constants. [5]

(d) Show that the torsion of a geodesic with nowhere vanishing curvature is equal to its geodesic torsion. [5]

(e) Give an example in each case of a surface of positive curvature, negative curvature and zero curvature. [2+2+1]

(f) Determine the asymptotic curves on the surface $\sigma(u, v) = (u\cos v,\ u\sin v, logu)$. [5]

(g) If $\gamma(t) = \sigma\big(u(t), v(t)\big)$ is a unit speed curve on a surface patch σ, then determine the relation among its curvature, normal curvature and geodesic curvature. [5]

(h) Prove that any tangent developable surface is isometric to a plane. [5]

2. Answer any three questions: 10×3 = 30

(a) Obtain a necessary and sufficient condition for a space curve to be a helix. [5+5]

(b) State and prove the fundamental theorem of a space curve. [2+8]

(c) Using Clairaut's theorem, determine the geodesics on the pseudosphere. [10]

(d) Deduce the Gaussian and mean curvatures of a right circular cylinder. [10]

(e) Prove that a connected surface of which every point is umbilic is either a part of plane or a part of a sphere. [10]

B.A./B.S.c 6th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH6DSE43
(Mechanics-II)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Deduce Galilean transformation from Newton's Second law of motion. [5]

(b) A circular area of radius 'a' is immersed in a homogeneous liquid with its plane vertical and centre at a depth 'h' ; find the depth of the centre of pressure. [5]

(c) (i) State Archimedes' Principle. [2]

(ii) A cone whose vertical angle is 2α, has the lowest generator horizontal and is filled with liquid, prove that the resultant pressure on the curved surface is $\sqrt{1+15\sin^2\alpha}$ times the weight of the liquid. [3]

(d) An equilateral triangular lamina suspended freely from **A,** rests with the side **AB** vertical and the side **AC** bisected by the surface of a heavy liquid. Prove that the [5] density of the lamina is to that of the liquid is **15 : 16 .**

(e) Prove that in a liquid at rest under the action of a force towards a fixed point, the surfaces of equal pressure are concentric spheres. [5]

(f) Discuss the Limitations of Newton's laws of motion in solving equations of motion. [5]

(g) (i) Define a holonomic constraint with example. [1+2]

(ii) Write down the equation of constraint for the motion of a solid sphere down an inclined plane. [2]

(h) Deduce the relation, [5]

$$\frac{T}{T_0} = 1 - \frac{\gamma - 1}{\gamma} \frac{z}{H} \ ,$$

assuming gravity to be constant.

2. Answer any three questions: 10×3 = 30

(a) (i) Define principal stresses and principal directions of stresses. [2+2]

(ii) The stress tensor at a point in a continuum is given by, [6]

$$(\tau_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$

Determine the principal stresses and the corresponding principal directions.

(b) (i) What is an adiabatic change of state? Derive the relation, pv^{γ} = constant for adiabatic expansion of a compressible fluid, where the symbols are to be explained by you. [2+3]

(ii) If the law connecting the pressure and density of the air is $p = k\rho^n$, prove that, neglecting variations of gravity and temperature, the height of the atmosphere would be $\frac{n}{n-1}$ times the homogeneous atmosphere. [5]

(c) (i) Establish the necessary condition for equilibrium of a fluid under the action of external forces whose components along the co-ordinate axes are **X, Y** and **Z .** [5]

(ii) A thin hollow cone with a base floats completely immersed in water whenever it is placed; show that the vertical angle is $2\sin^{-1}\frac{1}{3}$. [5]

(d) (i) Show that Kinetic Energy is not an invariant under Galilean Transformation but Acceleration remains invariant under this transformation. [5]

(ii) Discuss Gibbs-Appell's Principle of Least Constraint. [5]

(e) (i) Explain the concept of generalized co-ordinates in connection with fixing the configuration of a dynamical system. [3]

(ii) Establish Lagrange's equations of motion for a holonomic bilateral constraints. [5]

(iii) Is Largangian of a system unique? Justify your answer. [2]